

February 1, 2008

LBL-38047  
UCB-PTH-95/43 **$(S_3)^3$  flavor symmetry and  $p \rightarrow K^0 e^+ *$** Christopher D. Carone,<sup>1</sup> Lawrence J. Hall,<sup>1,2</sup> and Hitoshi Murayama<sup>1,2</sup><sup>1</sup> *Theoretical Physics Group**Lawrence Berkeley National Laboratory**University of California, Berkeley, California 94720*<sup>2</sup> *Department of Physics**University of California, Berkeley, California 94720***Abstract**

We show how to incorporate the lepton sector in a supersymmetric theory of flavor based on the discrete flavor group  $(S_3)^3$ . Assuming that all possible nonrenormalizable operators are generated at the Planck scale, we show that the transformation properties of the leptons and of the flavor-symmetry breaking fields are uniquely determined. We then demonstrate that the model has a viable phenomenology and makes one very striking prediction: the nucleon decays predominantly to  $Kl$  where  $l$  is a *first* generation lepton. We show that the modes  $n \rightarrow K^0 \bar{\nu}_e$ ,  $p \rightarrow K^+ \bar{\nu}_e$ , and  $p \rightarrow K^0 e^+$  occur at comparable rates, and could well be discovered simultaneously at the SuperKamiokande experiment.

---

\*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-90-21139.

## Disclaimer

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial products process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof, or The Regents of the University of California.

*Lawrence Berkeley Laboratory is an equal opportunity employer.*

# 1 Introduction

The origin of flavor has been a significant puzzle in particle physics since the discovery of the muon. The replication of fermion generations and the strongly hierarchical pattern of their masses and mixing angles is left unexplained in the Standard Model. In most theories that attempt to elucidate the puzzling features of flavor, new symmetries are introduced at mass scales that are large compared to the electroweak scale. The new scales may be associated with the breaking of flavor symmetries or of a grand unified gauge group. In either case, the introduction of a very high scale in the theory induces a large radiative correction to the Higgs mass squared, which destabilizes the hierarchy between the high scale and the electroweak scale. Supersymmetry is the most promising mechanism for avoiding this problem. Therefore, it is natural to consider the physics of flavor in the framework of the supersymmetric standard model (SSM).

Supersymmetry, however, complicates the problem of flavor by introducing a new sector of particles whose masses and mixing angles must also be understood. While no superpartner has yet been observed, the acceptable spectrum is constrained by low-energy processes. Most notably, a high degree of degeneracy is required among the light generation squarks to suppress dangerous flavor-changing effects [1], unless there is a strong alignment of quark and squark eigenstates [2, 3]. The challenge in a supersymmetric theory of flavor is to simultaneously explain both the suppression of flavor changing effects from the scalar sector and the hierarchical pattern of the quark Yukawa couplings. Flavor symmetries, spontaneously broken by a hierarchy of vacuum expectation values (vevs), provide an interesting tool for this purpose [4, 2]. However, there is considerable freedom in the choice of flavor group and symmetry breaking pattern.

In the recent literature, a number of authors have tried to meet this challenge by constructing models of flavor based on Abelian horizontal symmetries, often motivated by superstring theory [5]. However, many of these models have problems with large flavor-changing effects [6]. The only surviving explicit Abelian models of which we are aware are those of reference [2], which, however, rely on somewhat ad hoc sets of charge assignments to achieve alignment. Several authors have considered non-Abelian flavor

groups, leading to near degeneracy of the lightest two generation scalars, including  $SU(2)$  [4],  $O(2) \times U(1)$  [7] and  $U(2)$  [8], although the latter appears not to solve the flavor changing problem associated with the  $\epsilon$  parameter of  $K$  physics. A model has also been proposed in which the three generations of fermions are unified into an irreducible multiplet of a non-Abelian discrete symmetry [9]. This leads to sufficient squark degeneracy to solve the flavor changing problem, but a heavy top quark is not guaranteed, and results only by assuming that one of three Higgs fields remains light after flavor symmetry breaking.

Two of the authors (LJH and HM) have advocated the use of discrete, gauged non-Abelian family symmetries to obtain the desired degree of squark degeneracy [10]: global continuous symmetries are broken by quantum gravitational effects [11], while gauged continuous symmetries may generate  $D$ -term contributions to the squark and slepton masses that are nonuniversal [12], as in the model of reference [4]. It was demonstrated in Ref. [10] that the non-Abelian discrete group,  $(S_3)^3$ , is a promising choice for the flavor symmetry group of the SSM. The group  $S_3$  has both a doublet (**2**) and a non-trivial singlet representation (**1<sub>A</sub>**) into which the three generations of fermions can be embedded. In order to construct a viable model, three separate  $S_3$  factors are required, for the left-handed doublet fields  $Q$ , and the right-handed singlet fields  $U^c$  and  $D^c$ . The first and second generation fields transform as doublets, which ensures the degeneracy among the light generation squarks. The third generation fields must then transform as **1<sub>A</sub>**s so that the theory is free of discrete gauge anomalies. While the group  $S_3$  acts identically on three objects, the representation structure distinguishes between the generations. Thus, it is possible to choose the quantum numbers for the Higgs fields so that only the top Yukawa coupling is allowed in the symmetry limit. The hierarchical structure of Yukawa matrices can then be understood as a consequence of the sequential breaking of the flavor symmetry group. The model is appealing on more general grounds since discrete gauge symmetries arise naturally in superstring compactifications.

The model proposed in Ref. [10], however, did not address the problem of flavor in the lepton sector. The lepton Yukawa matrix is clearly hierarchical, and its eigenvalues are similar in size to those of the down quarks. In addition, a high degree of degeneracy between first and second generation

sleptons (or an alignment between slepton and lepton mass eigenstates) is required to suppress dangerous lepton flavor-violating processes. As far as this point is concerned, it is reasonable to expect that the flavor structure proposed in Ref. [10] for the quarks should work equally well when applied to the lepton sector. However, there is no guarantee that the flavor symmetry will provide an adequate suppression of the operators which mediate proton decay. Recall that there are two possible sources of proton decay in the SSM:  $R$ -parity violating dimension-four operators (like  $QDL$  or  $UDD$ ), and nonrenormalizable, dimension-five operators (like  $QQQL$  or  $UUDE$ ) which are likely to be generated at the Planck scale. In the first case, the coefficients of the  $R$ -parity violating operators are forced to be extremely small by the nonobservation of proton decay. The operators  $c_1 QsL$  and  $c_2 uds$ , for example, are constrained such that  $c_1 c_2 < O(10^{-26})$ . In this paper we simply assume that  $R$ -parity is an exact symmetry, and these operators are not present.<sup>†</sup> Assuming that quantum gravity effects violate global symmetries, however, non-renormalizable operators that conserve  $R$ -parity, but violate baryon and lepton number, are presumably generated at the Planck scale. The coefficients of the operators  $(Q_1 Q_{1,2}) Q_2 L_i / M_P$  are constrained to be no larger than  $O(\lambda^8)$  with  $\lambda \simeq 0.22$ . In the absence of other mechanisms to eliminate these operators,<sup>‡</sup> an adequate theory of flavor must explain why they are sufficiently suppressed after flavor symmetry-breaking effects are taken into account [13].

In this article, we extend the  $(S_3)^3$  model to the lepton sector. First, we require that the leptons transform under the same  $S_3$  flavor groups as the quarks. This is the simplest choice given that the ordinary Higgs fields transform nontrivially under  $S_3^Q$  and  $S_3^U$ . We select the transformation properties of the lepton fields under  $(S_3)^3$  so that we obtain the greatest similarity between the lepton and down-quark Yukawa matrices. The assignment must also forbid all dangerous dimension-five operators in the  $(S_3)^3$  symmetry

---

<sup>†</sup>Since matter parity is non-anomalous with the minimal SSM (MSSM) particle content, it can be considered as a discrete gauge symmetry, and is hence preserved by quantum gravity effects.  $R$ -parity is a product of matter parity and a  $2\pi$  rotation in the local Lorentz frame.

<sup>‡</sup>For example, we could impose a Peccei–Quinn symmetry [14], discrete symmetries [15], or gauge  $U(1)_B$  [16].

limit. We will see that the transformation properties of the lepton fields are uniquely fixed by these requirements. We then argue that the fundamental sources of flavor symmetry breaking are gauge singlet fields  $\phi$  that transform in the same way as the irreducible “blocks” of the quark Yukawa matrices. We will call these fields ‘flavons’ below. We show from a general operator analysis that models involving flavons with simpler transformation properties can all be excluded, if flavor physics originates at the Planck scale. With the flavor symmetry breaking originating only from the Yukawa matrices, we consider the contributions to lepton flavor violation and proton decay. We show that the model is consistent with the current experimental bounds. In addition, we show that the dominant proton decay modes in the  $(S_3)^3$  model are of the form  $p \rightarrow Kl$ , where  $l$  is a *first* generation lepton. This is never the case in either supersymmetric or non-supersymmetric grand unified theories. The prediction of these rather unique modes is exciting since the total decay rate is likely to be within the reach of the SuperKamiokande experiment.

In the next section we make the assumptions of our analysis explicit. In section 3, we introduce an economical form for the flavons  $\phi$ , which provide an adequate description of both lepton and quark mass matrices via dimension five interactions. We use flavor-changing and baryon number violating phenomenology to demonstrate that this choice is the simplest possible acceptable form for the flavons in section 5. This phenomenology is studied in much further detail in section 6, and conclusions are drawn in section 7.

## 2 The Framework

In this paper we construct a description of flavor, for both quarks and leptons, based on a flavor group  $(S_3)^3$ , spontaneously broken by a set of flavon fields,  $\phi$ . We write down an effective theory beneath the Planck scale in which the gauge symmetry is  $G_{SM} = SU(3) \times SU(2) \times U(1)$  and  $R$  parity is imposed. The field content is that of the minimal supersymmetric model, together with a set of flavon fields, which are necessarily all neutral under the gauge group since they take vevs much larger than the weak scale. The theory contains the most general  $F$  and  $D$  terms consistent with  $G_{SM} \times (S_3)^3 \times R_P$  with all interactions scaled by the appropriate powers of  $M_{Pl}$  and all dimensionless

coefficients of order unity. The single exception to this is the absence of a Planck scale mass for the Higgs doublets and for the flavons. In addition the theory is taken to possess supersymmetry breaking interactions which are the most general soft operators consistent with the symmetries of the theory. In particular, no universality assumptions are made to relate otherwise free parameters. At the renormalizable level, the theory is remarkably simple: the only  $F$  terms are the Yukawa coupling for the top quark, and possible trilinears amongst the flavon fields. The only supersymmetry breaking terms are: the three gaugino masses, a trilinear scalar interaction involving the top squarks coupled to a Higgs doublet, scalar masses for the flavons, the Higgs, and for the squarks and sleptons, which are diagonal in flavor space with the structure  $(m_1^2, m_1^2, m_3^2)$ . Supersymmetric non-renormalizable  $F$  terms lead to the Yukawa matrices becoming functions of  $\phi/M_{Pl}$ , while the non-renormalizable  $F$  and  $D$  terms, which contain supersymmetry breaking spurions, similarly lead to the trilinear  $A$  terms and the scalar masses becoming functions of  $\phi/M_{Pl}$ .

### 3 The Basic (2,2) Model

In the  $(S_3)^3$  model of Ref. [10], the quark chiral superfields  $Q$ ,  $U$ , and  $D$  are assigned to  $\mathbf{2} + \mathbf{1}_A$  representations of  $S_3^Q$ ,  $S_3^U$  and  $S_3^D$ , respectively. The first two generation fields are embedded in the doublet, for the reasons described in the Introduction. The Higgs fields both transform as  $(\mathbf{1}_A, \mathbf{1}_A, \mathbf{1}_S)$ 's, so that the top quark Yukawa coupling is invariant under the flavor symmetry group. The transformation properties of the Yukawa matrices are:

$$Y_U \sim \left( \frac{(\tilde{\mathbf{2}}, \tilde{\mathbf{2}}, \mathbf{1}_S)}{(\mathbf{1}_S, \tilde{\mathbf{2}}, \mathbf{1}_S)} \middle| \frac{(\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S)}{(\mathbf{1}_S, \mathbf{1}_S, \mathbf{1}_S)} \right) , \quad Y_D \sim \left( \frac{(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2})}{(\mathbf{1}_S, \mathbf{1}_A, \mathbf{2})} \middle| \frac{(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{1}_A)}{(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A)} \right) \quad (1)$$

where we use the notation  $\tilde{\mathbf{2}} \equiv \mathbf{2} \otimes \mathbf{1}_A$ ,<sup>§</sup>. Note that these matrices involve at most 7 irreducible multiplets of  $(S_3)^3$ . In Ref. [10],  $(S_3)^3$  was broken by only four types of flavons:  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S)$ ,  $\phi(\tilde{\mathbf{2}}, \tilde{\mathbf{2}}, \mathbf{1}_S)$ ,  $\phi(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A)$ , and  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2})$ , the minimal number which allows us to obtain realistic masses and mixings

---

<sup>§</sup> $\tilde{\mathbf{2}} = (a, b)$  is equivalent to  $\mathbf{2} = (b, -a)$ .

breaking	$S_3^Q$	$S_3^U$	$S_3^D$
$h_t V_{cb}$	$Z_2^Q$	$S_3^U$	$S_3^D$
$h_b$	$Z_2^Q$	$(S_3^U \times S_3^D)/Z_2$	
$h_t V_{ub}$	nothing	$(S_3^U \times S_3^D)/Z_2$	
$h_c, h_c \lambda$	nothing	$(Z_2^U \times S_3^D)/Z_2$	
$h_s, h_s \lambda$	nothing	$Z_2^{U,D}$	
$h_u, h_d$	nothing	nothing	nothing

Table 1: The sequential symmetry breaking pattern of  $(S_3)^3$  symmetry. The degeneracy between  $\tilde{d}$  and  $\tilde{s}$  is kept until the non-Abelian group  $(Z_2^U \times S_3^D)/Z_2$  is broken.  $Z_2^{U,D}$  symmetry remains unbroken until the last stage, which keeps up- and down-quarks massless.

[17]:

$$Y_U = \left( \begin{array}{cc|c} h_u & h_c \lambda & -h_t V_{ub} \\ 0 & h_c & -h_t V_{cb} \\ \hline 0 & 0 & h_t \end{array} \right), \quad Y_D = \left( \begin{array}{cc|c} h_d & h_s \lambda & 0 \\ 0 & h_s & 0 \\ \hline 0 & 0 & h_b \end{array} \right), \quad (2)$$

with  $\lambda \simeq 0.22$ . We will refer to this scenario as the basic (2,2) model below, because it involves flavons which transform as doublets under two of the  $S_3$  factors simultaneously. Note that it is not absolutely necessary that we generate  $V_{cb}$  and  $V_{ub}$  by rotations in the up sector, as we have indicated above. We can instead generate  $V_{cb}$  and  $V_{ub}$  in the down sector (by assigning appropriate values to the (1,3) and (2,3) elements in  $Y_D$ ) using the breaking parameter  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{1}_A)$  rather than  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S)$ . This choice generally gives us much weaker phenomenological constraints, as we will see later. In section 5 we study whether one can construct  $\phi(\tilde{\mathbf{2}}, \tilde{\mathbf{2}}, \mathbf{1}_S)$  and  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2})$  breaking parameters from products of flavons that transform as doublets under only one  $S_3$ , and find it is not possible within the framework specified in the previous section.

The form of the Yukawa matrices presented above can be understood as a consequence of the sequential breaking of the flavor symmetry, as shown in Table 1. Note that it is necessary to have different flavons associated with each step in the sequence, since all the components in a single irreducible multiplet typically become heavy at the same time unless a fine-tuning is



done. For instance, we assume that one chiral superfield transforming as  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S)$  acquires a vacuum expectation value (VEV) in its  $\mathbf{v}_1$  component, which breaks  $S_3^Q$  to  $Z_2^Q$  and generates  $h_t V_{cb}$ . A different chiral superfield transforming in the same way, remains light at this stage, but acquires a VEV in its  $\mathbf{v}_2$  component at lower scale to break  $Z_2^Q$  and generate  $h_t V_{ub}$ . Since  $S_3^Q$  is completely broken at this stage, a single chiral superfield  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2})$  is split into two  $(\mathbf{1}_A, \mathbf{2})$ s under the remaining  $(S_3^U \times S_3^D)/Z_2$  symmetry; both acquire VEVs in their  $\mathbf{v}_1$  components to break this symmetry down to  $Z_2^{U,D}$ . Since  $h_s$  and  $h_s V_{cd}$  are generated at the same stage of the symmetry breaking, there is a natural reason why the Cabibbo angle is rather large. The final stage of breaking is done by another  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2})$  to generate  $h_d$  (and by another  $\phi(\tilde{\mathbf{2}}, \tilde{\mathbf{2}}, \mathbf{1}_S)$  to generate  $h_u$  in the up sector). Therefore, the  $2 \times 2$  block in  $Y_D$  has the structure  $Y_D = Y_1 + Y_2$ , where

$$Y_1 = \begin{pmatrix} 0 & ah_s\lambda \\ 0 & h_s \end{pmatrix}, \quad Y_2 = \begin{pmatrix} h_d & 0 \\ 0 & 0 \end{pmatrix}. \quad (3)$$

Note that  $Y_1$  preserves  $Z_2^{U,D}$  symmetry. Of course the other elements in  $Y_2$  can be non-vanishing, but are expected to be of order  $h_d$  or less, and are irrelevant for our purposes. Similarly, the  $2 \times 2$  block in  $Y_U$  is given by  $Y_U = Y'_1 + Y'_2$ , where

$$Y'_1 = \begin{pmatrix} 0 & a'h_c\lambda \\ 0 & h_c \end{pmatrix}, \quad Y'_2 = \begin{pmatrix} h_u & 0 \\ 0 & 0 \end{pmatrix}. \quad (4)$$

Note that  $a$  and  $a'$  are order one constants, with  $a - a' = 1$ .

## 4 Incorporating Lepton Sector

The Higgs fields in the  $(S_3)^3$  model transform nontrivially under the flavor symmetry group. Since the lepton fields acquire their masses in the MSSM from the same Higgs fields as the quarks, the leptons should transform under the same  $(S_3)^3$  flavor symmetry. We are led by three principles in determining the precise transformation properties of the lepton fields:

1. We do not allow any new flavor symmetries (e.g. new  $S_3$  factors) that arise only in the lepton sector. The only flavor symmetry in the theory is  $S_3^Q \times S_3^U \times S_3^D$ .

2. We assign the transformation properties of the lepton fields so that the charged lepton Yukawa matrix is similar to that of the down quarks. This choice is suggested by the phenomenology.
3. We require that the most dangerous dimension-five operator that contributes to proton decay,  $(QQ)(QL)$ , is forbidden in the  $(S_3)^3$  symmetry limit.

As we will see below, these principles are sufficient to completely determine the transformation properties of the lepton fields.

Let us first consider the consequences of the first two conditions. The down-quark Yukawa matrix is a coupling between the left-handed quark fields  $Q \sim (\mathbf{1}_A + \mathbf{2}, \mathbf{1}_S, \mathbf{1}_S)$  and the right-handed down quark fields  $D \sim (\mathbf{1}_S, \mathbf{1}_S, \mathbf{1}_A + \mathbf{2})$ . We know that the Yukawa matrix of the charged leptons is quite similar to that of the down quarks, up to factors of order three [18] at high scales:

$$m_b \simeq m_\tau, \quad m_s \simeq \frac{1}{3}m_\mu, \quad m_d \simeq 3m_e. \quad (5)$$

Therefore, we look for an assignment of lepton transformation properties that leads automatically to this observed similarity. There are only two possibilities:

$$\begin{array}{c|ccc} & S_3^Q & S_3^U & S_3^D \\ \hline L & \mathbf{1}_A + \mathbf{2} & \mathbf{1}_S & \mathbf{1}_S \\ E & \mathbf{1}_S & \mathbf{1}_S & \mathbf{1}_A + \mathbf{2} \end{array} \quad \text{or} \quad \begin{array}{c|ccc} & S_3^Q & S_3^U & S_3^D \\ \hline L & \mathbf{1}_S & \mathbf{1}_S & \mathbf{1}_A + \mathbf{2} \\ E & \mathbf{1}_A + \mathbf{2} & \mathbf{1}_S & \mathbf{1}_S \end{array} . \quad (6)$$

The third condition above allows us to distinguish between these two alternatives. In the first assignment, the operator  $(Q_i Q_i)(Q_j L_j)$  is allowed by the  $(S_3)^3$  symmetry, and we have proton decay at an unacceptable rate. Therefore, only the second assignment in Eq. (6) satisfies all three criteria listed above. We could have obtained the same conclusion by considering the  $UUDE$  operators as well.

The remaining question that we need to answer is how the factors of three in (5) enter in the Yukawa matrices. One plausible explanation is that they originate from fluctuations in the order one coefficients that multiply the  $(S_3)^3$  breaking parameters which generate the quark and lepton Yukawa matrices. As discussed in the previous section, it is quite likely that the

$2 \times 2$  block in  $Y_d$  is generated by two  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2})$  breaking parameters. An acceptable lepton Yukawa matrix is obtained by allowing coefficients in both breaking parameters to deviate from unity by a factor of three. Throughout this paper we take  $Y_l = 3Y_1 + \frac{1}{3}Y_2$ .

## 5 Uniqueness

In the basic (2,2) model introduced in the previous section, the quark Yukawa matrices were taken as the only sources of flavor symmetry breaking. Thus, the Higgs fields that spontaneously break the flavor symmetry group in the corresponding high energy theory would come in exactly four representations of  $(S_3)^3$ :  $(\mathbf{2}, \mathbf{1}_A, \mathbf{2})$ ,  $(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A)$ ,  $(\mathbf{2}, \mathbf{2}, \mathbf{1}_S)$ , and  $(\mathbf{2}, \mathbf{1}_S, \mathbf{1}_S)$ . These correspond to the various blocks of the quark Yukawa matrices. We have assumed that no other representations are involved in  $(S_3)^3$  breaking in the full theory.

The issue that remains to be addressed is whether this picture of the high energy theory is overly restrictive. There are  $3^3 - 1 = 26$  nontrivial representations of  $(S_3)^3$  that we could have used had we tried to build an adequate high-energy theory of flavor symmetry breaking at the start. There is no reason to assume *a priori* that a model cannot be constructed with a different, more fundamental set of symmetry breaking parameters. What we will show in this section is that there are in fact no simple models of  $(S_3)^3$  breaking involving symmetry breaking parameters that are more fundamental than the ones adopted in the basic (2,2) model. We will systematically exclude all the reasonable alternatives. While the basic (2,2) model served as an existence proof for a successful  $(S_3)^3$  model in Ref. [10], we will show here that the choice of symmetry breaking parameters in this model is in fact unique.

We first would like to consider the class of models in which there are no “(2,2)” representations, i.e., there are no fields that transform as doublets under more than one  $S_3$  group at a time. As a starting point, let us consider a simple toy model that illustrates the phenomenological problems common to models of this type. The flavon content is

$$H_Q^{(i)} \sim (\mathbf{2}, \mathbf{1}_S, \mathbf{1}_S) \quad H_U \sim (\mathbf{1}_S, \mathbf{2}, \mathbf{1}_S) \quad H_D \sim (\mathbf{1}_S, \mathbf{1}_S, \mathbf{2}) \quad (7)$$

where  $i = 1 \dots 2$  labels two distinct doublets, and

$$\chi_1 \sim (\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_S) \quad \chi_2 \sim (\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A) \quad \chi_3 \sim (\mathbf{1}_A, \mathbf{1}_A, \mathbf{1}_S) \quad \chi_4 \sim (\mathbf{1}_A, \mathbf{1}_A, \mathbf{1}_A) \quad (8)$$

The doublet  $H$  fields were chosen to have the simplest quantum number assignments possible. The quantum number assignments of the  $\chi$  fields were chosen for a variety of reasons, that will become clear in context below. When the flavon fields acquire vevs, the various blocks of the quark mass matrices are generated from higher dimension operators. The way in which this model fails is instructive and will simplify the discussion of the other models to follow, so we will proceed in some detail.

The two-by-two quark Yukawa matrices in this model are obtained by taking the products  $\sigma_2 H_Q H_U^T \sigma_2^T$  and  $\sigma_2 H_Q H_D \chi_U$ , for the up and down sectors respectively. Given this construction, we require two  $H_Q$  fields in order to assure a nonvanishing Cabibbo angle. Let us denote the ratio of the vevs of the  $H$  and  $\chi$  fields to an appropriate cutoff scale by  $\epsilon$  and  $\delta$ , respectively. If we take the  $H_Q$  and  $H_D$  to be of the form

$$H_Q^{(i)} = \epsilon_Q \begin{bmatrix} 1 \\ \lambda \end{bmatrix} \quad H_D = \epsilon_D \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \quad (9)$$

then we obtain the two-by-two down Yukawa matrix

$$\epsilon_Q \epsilon_D \delta_1 \begin{bmatrix} \lambda^2 & \lambda \\ \lambda & 1 \end{bmatrix} \quad (10)$$

where  $\lambda \approx 0.22$  is the Cabibbo angle. By setting the combination  $\epsilon_Q \epsilon_D \delta_1 \sim \lambda^5$ , we obtain the correct strange quark Yukawa coupling, assuming  $\tan \beta \sim 1$ .

The down quark Yukawa coupling has not yet been generated, however, because the matrix above has a zero eigenvalue. This is a generic feature of all models in which the down-strange Yukawa matrix is formed by taking the product of two doublets. Of course, the same problem arises in the up-charm Yukawa matrix as well. The way in which the up and down Yukawa couplings are generated is through other operators that are of higher order in the symmetry breaking. Notice that we can obtain a correction to the down-strange Yukawa matrix via the operator  $\chi_4 H_Q H_D^T \sigma_2^T$  which is of the

form

$$\epsilon_Q \epsilon_D \delta_4 \begin{bmatrix} 1 & \lambda \\ \lambda & \lambda^2 \end{bmatrix} \quad (11)$$

If we take  $\delta_4/\delta_2 \sim \lambda^2$ , then we obtain a correction to the (1,1) component of (10). This is sufficient to lift the zero eigenvalue at order  $\epsilon_Q \epsilon_D \delta_4 \sim \lambda^7$ , which is of the correct order in  $\lambda$  to generate the down Yukawa coupling. A similar mechanism occurs in the up sector as well. If we take

$$H_U^{(i)} = \epsilon_Q \begin{bmatrix} 1 \\ \lambda \end{bmatrix} \quad (12)$$

then the up-charm Yukawa matrix  $\sigma_2 H_Q H_U^T \sigma_2^T$  is given by

$$\epsilon_Q \epsilon_U \begin{bmatrix} \lambda^2 & \lambda \\ \lambda & 1 \end{bmatrix}$$

We take  $\epsilon_Q \epsilon_U \sim \lambda^4$  in order to generate the correct charm quark Yukawa coupling. The zero eigenvalue in this matrix is lifted via the operator  $\chi_3 H_Q H_U^T$ , which is of the form

$$\epsilon_Q \epsilon_U \delta_3 \begin{bmatrix} 1 & \lambda \\ \lambda & \lambda^2 \end{bmatrix}$$

If we take  $\delta_3 \sim \lambda^4$ , then this corrects the (1,1) entry of the first operator, lifting the zero eigenvalue at order  $\lambda^8$ , as desired.

The inescapable problems with the model above arise from flavor changing neutral currents and proton decay considerations. In the first case, the off-diagonal elements in the squark mass matrices can be constructed at first order in the symmetry breaking  $\epsilon$  parameters via the ‘ $2^3$  invariant’

$$(\tilde{Q}^{*T} \sigma_3 \tilde{Q}) H_1 - (\tilde{Q}^{*T} \sigma_1 \tilde{Q}) H_2 \quad (13)$$

Here  $\tilde{Q}$  is the squark doublet that transforms under the same  $S_3$  group as  $H$ , and we use subscripts to denote the components of the  $H$  doublet. The resulting off-diagonal elements are constrained by flavor changing neutral currents processes, such that  $\langle H_{Q2} \rangle < 0.05$ ,  $\langle H_{D2} \rangle < 0.05$ , and  $\sqrt{\langle H_{Q2} \rangle \langle H_{D2} \rangle} < 0.006$ . Given the form of the  $H$  fields described above, this implies that  $\sqrt{\epsilon_Q \epsilon_D \lambda} < 0.006$ . However, we saw earlier that the strange quark Yukawa coupling  $h_s \sim \epsilon_Q \epsilon_D \delta_1$  is of order  $\lambda^5$ . Hence we require  $\delta_1 \approx 3$ , which contradicts our assumption that all the  $\epsilon$ ’s and  $\delta$ ’s are small symmetry breaking

parameters. In other words, for  $\delta_1 < 1$ , we cannot generate a large enough strange quark Yukawa coupling if we are to simultaneously satisfy the FCNC constraints.

The second problem is that there are operators in this model that contribute to proton decay at an unacceptable level. We can construct the representation  $(\mathbf{2}, \mathbf{1}_S, \mathbf{2})$  at order  $\epsilon_Q \epsilon_D$ , which contributes to proton decay via the operator  $\frac{1}{M}(QQQL)$  at the same order. This forces us to take  $\epsilon_Q \epsilon_D < \lambda^7$ , which again makes it impossible to generate a large enough strange quark Yukawa coupling.

What we have found is that it is impossible to generate large enough down quark Yukawa couplings when the  $H$  fields have the simple transformation properties described above. One way to remedy this problem is to allow these fields to transform under more than one  $S_3$  group, so that the contributions to the off-diagonal elements of the corresponding squark mass matrices occur at higher order, and dangerous proton decay operators are sufficiently suppressed. The minimal modification of the model above with this property is a model in which some or all of the  $H$  fields transform as  $\mathbf{1}_A$ 's under an additional (or both remaining)  $S_3$  groups. Since it is possible in this case to build the down-strange Yukawa matrix from the product of  $H_Q$  and  $H_D$  alone, we will restrict our discussion to models in which the both the down-strange and up-charm Yukawa matrices are generated at second order in the symmetry-breaking  $\epsilon$  parameters. Any model in which these matrices are generated at higher order in the symmetry breaking will involve the representations described below as composite operators, and at the very least will be subject to the same constraints. There are a finite number of possibilities for the type of model of interest, and we will now outline why each fails:

*case 1:*  $H_D \sim (\mathbf{1}_A, \mathbf{1}_S, \mathbf{2})$ . This representation contributes to proton decay through the operator  $\frac{1}{M}(QQQL)$  at order  $\lambda^2 \epsilon_D$ . Assuming that none of the  $\epsilon$ 's are larger than order  $\lambda$ , then  $\epsilon_D \geq \lambda^4$  so that we can generate a large enough strange quark Yukawa coupling. Hence, the coefficient of the proton decay operator is order  $\lambda^6$  or greater. This leads to an enhancement in the proton decay rate by four orders of magnitude over that of the basic (2,2) model, and this possibility is excluded.

*case 2:*  $H_D \sim (\mathbf{1}_S, \mathbf{1}_A, \mathbf{2})$ . To construct the upper-left two by two block

of the down quark mass matrix using this representation we must also have  $H_Q^{(i)} \sim (\mathbf{2}, \mathbf{1}_S, \mathbf{1}_S)$ . (The alternative choice  $H_Q^{(i)} \sim (\mathbf{2}, \mathbf{1}_S, \mathbf{1}_A)$ , for any  $i$ , is excluded by the proton decay constraints, as we will discuss later.) Notice that we generate the (1,3) and (2,3) entries of the down quark matrix from the product of  $H_Q$  and  $h_b$ . Thus we require  $\epsilon_Q$  to be order  $\lambda^2$  or smaller if  $V_{ub}$  and  $V_{cb}$  are not to be unacceptably large. This forces us to take  $\epsilon_D$  of order  $\lambda^3$  or larger if we are to generate an adequate strange quark Yukawa coupling. Now the problem arises because  $H_D$  also contributes to the (3,1) and (3,2) entries of the down quark Yukawa matrix at order  $\epsilon_D$ , which we have just argued is of order  $\lambda^3$  or larger. This gives us at least an order  $\lambda$  rotation on the right-handed down quark fields between the first and third generations. In the basis where the quark Yukawa matrices are diagonal, this yields a (3,1) entry in the right-handed down squark mass matrix of order  $\lambda M_1$  (Recall that the diagonal components of the squark mass matrices  $M_1$  and  $M_3$  are unconstrained.) This is in marginal disagreement with the the bound  $(m^2)_{13}^D/M_1^2 < 0.1$  from the  $B^0-\overline{B}^0$  constraint. A more serious problem arises because there is also an order 1 rotation between the right-handed  $b$  and  $s$  fields, which spoils the degeneracy between the second and first generation squarks; since there is an order  $\lambda$  rotation on the right-handed squarks of the first two generations in this model, we obtain a  $(\delta_{RR}^d)_{12}$  of order  $\lambda$ , which exceeds the bound  $(\delta_{RR}^d)_{12} < 0.05$ .

*case 3:*  $H_D \sim (\mathbf{1}_A, \mathbf{1}_A, \mathbf{2})$  In models that include this representation we must also have  $H_Q^{(i)} \sim (\mathbf{2}, \mathbf{1}_S, \mathbf{1}_S)$ . (Again, the alternative  $H_Q^{(i)} \sim (\mathbf{2}, \mathbf{1}_S, \mathbf{1}_A)$ , for any  $i$ , is excluded by proton decay constraints, as discussed below.)  $H_Q$  contributes to the (1,3) and (2,3) entries of both the up and down Yukawa matrices at order  $\epsilon_Q$  and  $\epsilon_Q h_b$  respectively, but always with the largest component in the (3,1) entry. Thus, this model predicts  $V_{ub} > V_{cb}$ , and is therefore excluded.

Now that we have established that  $H_D \sim (\mathbf{1}_S, \mathbf{1}_S, \mathbf{2})$  is the only viable representation for  $H_D$  (that has no more than 2 degrees of freedom) we are forced to modify the  $H_Q$  if we are going to evade the problems of the simple model presented at the beginning of this section.

*case 4:*  $H_Q \sim (\mathbf{2}, \mathbf{1}_A, \mathbf{1}_A)$ . This representation contributes at order  $\epsilon_Q$  to the (1,3) and (2,3) entries of the down quark Yukawa matrix. Thus,  $\epsilon_Q$  can be no larger than order  $\lambda^5$  if  $V_{cb}$  and  $V_{ub}$  are to be small enough. In this case,

we cannot generate a large enough strange quark Yukawa coupling.

*case 5:*  $H_Q \sim (\mathbf{2}, \mathbf{1}_S, \mathbf{1}_A)$ . This contributes to proton decay via the  $\frac{1}{M}(QQQL)$  operator at order  $\epsilon_Q$  and is immediately excluded.

*case 6:*  $H_Q \sim (\mathbf{2}, \mathbf{1}_A, \mathbf{1}_S)$ . We can construct the spurion  $(\mathbf{2}, \mathbf{1}_S, \mathbf{1}_A)$  at order  $h_b \epsilon_Q \sim \lambda^3 \epsilon_Q$ , which contributes to proton decay via  $\frac{1}{M}(QQQL)$  at the same order. This forces us to take  $\epsilon_Q \sim \lambda^4$  or smaller, which in turn tells us that  $\epsilon_D \sim \lambda$ , in order to generate a large enough strange quark Yukawa coupling. However, this is excluded by the flavor changing neutral current constraints for  $H_D \sim (\mathbf{1}_S, \mathbf{1}_S, \mathbf{2})$ .

The discussion above forces us to consider representations for the  $H$ 's that have four degrees of freedom. Let us consider the following possibilities:

*case 7:*  $H_D \sim (\mathbf{2}, \mathbf{1}_S, \mathbf{2})$ . We could imagine constructing a model in which the down two by two yukawa matrix is a product of  $(\mathbf{2}, \mathbf{1}_S, \mathbf{2})$  and  $h_b$ . However,  $H_D$  now gives us a contribution to proton decay at order  $\epsilon_D$ , and hence this alternative is excluded.

*case 8:* Other Models without a  $(\mathbf{2}, \mathbf{1}_A, \mathbf{2})$ . We still might hope to construct models without a  $(\mathbf{2}, \mathbf{1}_A, \mathbf{2})$  if we can generate the upper two by two block of the down quark Yukawa matrix at second order in the symmetry breaking. In such models, we require that the upper two by two block of the up quark Yukawa matrix be generated at second order as well. This restricts us to models with the representations (a)  $(\mathbf{1}_A, \mathbf{2}, \mathbf{2})$  and  $(\mathbf{2}, \mathbf{2}, \mathbf{1}_A)$  or (b)  $(\mathbf{1}_S, \mathbf{2}, \mathbf{2})$  and  $(\mathbf{2}, \mathbf{2}, \mathbf{1}_A)$ . In both cases, the down two by two block is generated by taking the product of the two spurions, while the up matrix is generated by taking the product of  $(\mathbf{2}, \mathbf{2}, \mathbf{1}_A)$  and  $h_b$ . Of course, we must also introduce a  $(\mathbf{2}, \mathbf{1}_S, \mathbf{1}_S)$  if we are to generate sufficient  $V_{ub}$  and  $V_{cb}$ . The problem with these models is that we can also construct a  $(\mathbf{2}, \mathbf{1}_S, \mathbf{2})$  by taking a product of the same spurions that generate the strange quark Yukawa coupling. Hence, there is a contribution to proton decay at order  $\lambda^5$ , and these models are excluded.

We have seen above that we must have a  $(\mathbf{2}, \mathbf{1}_A, \mathbf{2})$  as a fundamental field in the model, and that the down and strange Yukawa couplings are generated at first order in the symmetry breaking. If we require that the up quark two by two block also be generated at first order (so that the symmetry breaking is of a comparable magnitude) then we must introduce a fundamental  $(\mathbf{2}, \mathbf{2}, \mathbf{1}_s)$ . Since we cannot generate large enough values for  $V_{ub}$  and



$V_{cb}$  by taking products of these representations alone, we must also introduce a  $(\mathbf{2}, \mathbf{1}_s, \mathbf{1}_s)$ . Finally, we generate  $h_b$  most economically with a  $(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A)$ . Thus, we have arrived uniquely at the basic (2,2) model described in sections 3 and 4.

## 6 Phenomenology

In the previous sections, we showed that the transformation properties of both the lepton and flavon fields are uniquely determined in the  $(S_3)^3$  model, if all possible nonrenormalizable operators are generated at the Planck scale. With these results at hand, we may now consider lepton flavor violation and proton decay, two topics that depend crucially on the extension of the model to the lepton sector. We will also consider the bounds from CP-violating processes, which were not covered in the original paper. We demonstrate that the model is indeed viable phenomenologically. (Whenever we discuss numerical estimates, we take a representative choice  $\tan \beta \simeq 2$ .) In addition, we show that the model predicts the dominance of the  $Ke$  proton decay mode over the  $K\mu$  or  $\pi e$  mode, which is in sharp contrast to the situation in supersymmetric or non-supersymmetric GUTs.

### 6.1 Lepton Flavor Violation

The strongest constraint on lepton flavor violation comes from the non-observation of the  $\mu \rightarrow e\gamma$  decay mode. In our model, the contribution of the off-diagonal term in the purely left-handed slepton mass matrix (the LL matrix) is small enough ( $\sim h_s^2 \lambda \sim 1 \times 10^{-7}$ ) to avoid the experimental constraint for any value of  $m_{\tilde{l}}$  above the LEP bound. The stringent limits come from the purely right-handed slepton mass matrix (RR) and the left-right (LR) matrix, which we discuss in this section. We follow the notation of Barbieri, Hall and Strumia [19] below. For simplicity, we work in the approximation where the exchanged neutralino is a pure bino state.

The one-loop slepton and bino exchange diagram that picks up the off-diagonal (2,1) component in LR mass matrix generates the operator

$$\mathcal{O} = \frac{e}{2} F_2(m_R^2, m_L^2, M_1^2) \bar{e}_R i \sigma^{\mu\nu} \mu_L F_{\mu\nu}, \quad (14)$$

where  $F_{\mu\nu}$  is the electromagnetic field strength and

$$F_2(m_R^2, m_L^2, M_1^2) = \frac{\alpha_Y}{4\pi} (m_{LR}^2)_{21} \frac{G_2(m_R^2, M_1^2) - G_2(m_L^2, M_1^2)}{m_R^2 - m_L^2}, \quad (15)$$

with

$$G_2(m^2, M^2) = \frac{M}{2(m^2 - M^2)^3} \left[ m^4 - M^4 - 2m^2 M^2 \ln \left( \frac{m^2}{M^2} \right) \right]. \quad (16)$$

The decay width is given by

$$\Gamma(\mu \rightarrow e\gamma) = \frac{\alpha}{4} m_\mu^3 |F_2|^2, \quad (17)$$

and the bound  $\text{Br}(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$  implies  $|F_2| < 2.6 \times 10^{-12} \text{ GeV}^{-1}$ . In order to compare this bound to the prediction of our model, let us take  $m_R = m_L = m = 300 \text{ GeV}$  and  $M_1 = 100 \text{ GeV}$  as a representative case. Note that  $F_2$  is at its maximum around  $M_1 \sim m/2$ , and our choice gives almost the largest possible  $F_2$  for fixed  $m$ . We obtain

$$\frac{(m_{LR}^2)_{21}}{m^2} < 1.0 \times 10^{-5} \quad (18)$$

for this choice of parameters. In our model, the (2,2) and (1,2) elements in  $Y_l$  belong to the same irreducible multiplet, and diagonalization of  $Y_l$  also diagonalizes LR mass matrix at  $O(h_s \lambda)$ . The term which may not be simultaneously diagonalizable comes from the piece  $Y_2 \sim h_d$ , and hence

$$(m_{LR}^2)_{21} \sim m_d \lambda A, \quad (19)$$

where  $m_d$  is the down quark mass evaluated at the Planck scale  $m_d \simeq 10 \text{ MeV}/3$ , and  $A$  is a typical trilinear coupling. If we take  $A \sim 100 \text{ GeV}$ , then  $(m_{LR}^2)_{21}/m^2 \sim 0.8 \times 10^{-6}$  and the constraint (18) is easily satisfied. Since we have consistently allowed an order 3 ambiguity in estimating the coefficients of various operators, and we do not know the precise value of  $m_d$ , the actual constraint is weaker than the one considered above. The (1,2) element in the LR mass matrix contributes in exactly the same way as the (2,1) entry, except that the chiralities of the electron and muon in eq. (14) are flipped. Hence, the (1,2) element is subject to the same constraint, which again is clearly satisfied in our model.

The RR mass matrix also contributes to the operator in (14). In this case, the function  $F_2$  is given by

$$F_2 = \frac{\alpha_Y}{4\pi} m_\mu (m_{RR}^2)_{12} \frac{\partial G_1(m^2, M_1^2)}{\partial m^2}, \quad (20)$$

where

$$\frac{\partial G_1(m^2, M_1^2)}{\partial m^2} = \frac{1}{6(m^2 - M^2)^5} \times \left( m^6 - 9m^4 M^2 - 9m^2 M^4 + 17M^6 + (18m^2 M^4 + 6M^6) \log\left(\frac{m^2}{M^2}\right) \right). \quad (21)$$

This is a monotonically decreasing function of  $M$  for fixed  $m$ . For the bino and slepton masses chosen earlier, we obtain the bound

$$\frac{(m_{RR}^2)_{12}}{m^2} < 0.023, \quad (22)$$

while in our model

$$(m_{RR}^2)_{12}/m^2 \simeq h_t V_{cb} \lambda \sim 0.009. \quad (23)$$

The bound on the (1,2) element of the RR matrix is easily satisfied in our model. Note had we chosen the option of generating  $V_{cb}$  and  $V_{ub}$  in the down sector, as discussed in section 3, we would have obtained a much smaller off-diagonal element  $(m_{RR}^2)_{12}$ . In this case the breaking parameter transforms as  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{1}_A)$  rather than  $\phi(\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S)$ , and does not contribute to the RR mass matrix at the first order in  $\phi$ . The leading contribution is then

$$(m_{RR}^2)_{12}/m^2 \simeq h_b^2 V_{cb} \lambda \sim 3 \times 10^{-6}, \quad (24)$$

and the constraint (22) is again easily satisfied.

Finally, there is another contribution to  $\mu \rightarrow e\gamma$  from the mixing to the third generation sleptons. Since the third generation scalars can be non-degenerate with the first and second generation ones, GIM cancellation does not occur. This is similar to the situation in the minimal SO(10) model [20, 19], where the third generation sleptons are much lighter than the others, while the rotations to the quark mass eigenstate basis are CKM-like. Recall that in the minimal SO(10) model, the flavor changing factors in the amplitudes are  $V_{td}V_{ts}$ , whereas in our model they are  $V_{ub}V_{cb}$ , which is about a factor of three smaller.

To summarize, the  $(S_3)^3$  model is clearly consistent with the experimental bounds on lepton flavor violation, and predicts  $\mu \rightarrow e\gamma$  at a rate just beyond the current limit.

## 6.2 CP Violation

If we allow arbitrary phases in the symmetry breaking parameters, the squark mass matrices can have complex elements which contribute to CP violating effects. The most stringent limit on these phases comes from  $\epsilon'$ , and has been studied by Gabrielli, Masiero and Silvestrini [21]. If we consider the most extreme case imaginable, where all the off-diagonal elements of the down squark mass matrices in our model are purely imaginary, then we find

$$\text{Im} \left( \frac{(m_{LR}^2)_{12}}{m^2} \right) \simeq h_d \lambda \simeq 9 \times 10^{-6}, \quad (25)$$

$$\text{Im} \left( \frac{(m_{LL}^2)_{12}}{m^2} \right) \simeq h_t V_{cb} \lambda \simeq 0.009, \quad (26)$$

$$\text{Im} \left( \frac{(m_{RR}^2)_{12}}{m^2} \right) \simeq h_s^2 \lambda \simeq 1 \times 10^{-7}. \quad (27)$$

in the basis where the quark Yukawa matrices are diagonal. The constraints on these elements are

$$\left| \text{Im} \left( \frac{(m_{LR}^2)_{12}}{m^2} \right) \right| < 2 \times 10^{-5}, \quad (28)$$

$$\sqrt{\text{Im} \left( \frac{(m_{LL,RR}^2)_{12}}{m^2} \right)^2} < 3 \times 10^{-3}, \quad (29)$$

$$\sqrt{\text{Im} \left( \frac{(m_{RR}^2)_{12}}{m^2} \frac{(m_{LL}^2)_{12}}{m^2} \right)} < 2 \times 10^{-4}, \quad (30)$$

for  $m_{\tilde{q}} \simeq m_{\tilde{g}} \simeq 500$  GeV. One can see that all constraints are easily satisfied. In addition, there are a number of factors that make the actual bounds on our model weaker: (1) This analysis is valid only up to the unknown factors of order one that multiply the symmetry breaking operators. (2) The renormalization group running of soft SUSY breaking masses always tends to make the diagonal elements in the LL, RR, and LR matrices larger at lower energies, so that the true constraints on our model are generally weaker than those given above. (3) The elements  $V_{cb}$  and  $V_{ub}$  could instead be generated in the down sector, in which case  $(m_{LL}^2)_{12}/m^2 \simeq h_b^2 V_{cb} \lambda$ , and the SUSY contribution to  $\epsilon'$  becomes negligible. (4) We have no reason to expect that all the off-diagonal elements of the squark mass matrices in our model are purely imaginary, as we have assumed to obtain these bounds.

operator	$(S_3)^3$ representation	largest coefficient
$(Q_3 Q_3)(Q_i L_3)$	$(\mathbf{2}, \mathbf{1}_S, \mathbf{1}_A)$	$h_s h_d$ or $h_b h_c h_u$
$(Q_3 Q_3)(Q_i L_i)$	$(\mathbf{2}, \mathbf{1}_S, \mathbf{2})$	$h_b h_s$
$(Q_i Q_i)(Q_3 L_i)$	$(\mathbf{2}, \mathbf{1}_S, \mathbf{2})$	$h_b h_s$
$(Q_i Q_i)(Q_3 L_3)$	$(\mathbf{1}_A, \mathbf{1}_S, \mathbf{2})$	$h_t h_b h_s \lambda V_{cb}$
$(Q_i Q_i)(Q_3 L_3)$	$(\mathbf{2}, \mathbf{1}_S, \mathbf{1}_A)$	$h_s h_d$ or $h_b h_c h_u$
$(Q_i Q_i)(Q_3 L_3)$	$(\mathbf{1}_A, \mathbf{1}_S, \mathbf{1}_A)$	$h_s h_d$
$(Q_i Q_i)(Q_i L_3)$	$(\mathbf{2}, \mathbf{1}_S, \mathbf{1}_A)$	$h_s h_d$ or $h_b h_c h_u$
$(Q_i Q_i)(Q_i L_i)$	$(\mathbf{2}, \mathbf{1}_S, \mathbf{2})$	$h_b h_s$

Table 2: A complete list of possible baryon-number violating dimension-five operators involving doublet fields. All other operators vanish because of symmetry reasons.  $Q_i$  and  $L_i$  generically refer to either first- or second-generation fields, while  $Q_3$  and  $L_3$  refer to third-generation fields. For operators that are multiplets under the  $(S_3)^3$  symmetry, the coefficient of the largest component is given.

As in the case of  $\mu \rightarrow e\gamma$ , there is also a contribution to  $\epsilon'$  from the third generation squarks, similar to the minimal SO(10) case [20]. This falls within an acceptable range. Hence, it is reasonable to conclude that the model is consistent with the observed CP violating phenomenology.

### 6.3 Proton Decay

Since we have assumed throughout this paper that all possible nonrenormalizable operators are generated at the Planck scale, the task of studying proton decay in our model is a simple one. We first write down all the possible dimension-five operators that contribute to proton decay and identify their transformation properties under  $(S_3)^3$ . The coefficients can be estimated as the product of Yukawa couplings that will produce the desired symmetry breaking effect. A list of possible operators and their coefficients is given in Tables 2 and 3.

As we can see from the tables, the most important operator involving left-handed fields is  $(Q_i Q_i)(Q_i L_i)/M_*$ , where  $M_* \equiv M_{Pl}/\sqrt{8\pi}$  is the reduced Planck mass, and where parentheses indicate a contraction of SU(2) indices. This operator transforms as a  $(\mathbf{2}, \mathbf{1}_S, \mathbf{2})$  under  $(S_3)^3$ , and therefore has a coef-

operator	$(S_3)^3$ representation	largest coefficient
$tu_i b \tau$	$(\mathbf{1}_A, \mathbf{2}, \mathbf{1}_A)$	$h_t h_b h_c V_{ub}$
$tu_i b e_i$	$(\mathbf{2}, \mathbf{2}, \mathbf{1}_A)$	$h_b h_c$
$tu_i d_i \tau$	$(\mathbf{1}_A, \mathbf{2}, \mathbf{2})$	$h_c h_s \lambda$
$tu_i d_i e_i$	$(\mathbf{2}, \mathbf{2}, \mathbf{2})$	$h_c h_s$
$uc b \tau$	$(\mathbf{1}_A, \mathbf{1}_A, \mathbf{1}_A)$	$h_t^2 h_b V_{cb} V_{ub}$
$uc b e_i$	$(\mathbf{2}, \mathbf{1}_A, \mathbf{1}_A)$	$h_t h_b V_{cb}$
$uc d_i \tau$	$(\mathbf{1}_A, \mathbf{1}_A, \mathbf{2})$	$h_t h_s V_{cb} \lambda$
$uc d_i e_i$	$(\mathbf{2}, \mathbf{1}_A, \mathbf{2})$	$h_s$

Table 3: A complete list of possible baryon-number violating dimension-five operators involving singlet fields. All other operators vanish because of symmetry reasons.  $u_i$ ,  $d_i$ , and  $e_i$  generically refer to either first- or second-generation fields, while  $t$ ,  $b$  and  $\tau$  refer to the third-generation fields. The only possible product of the form  $u_i u_i$  is  $uc$  because of the anti-symmetry in the color indices. For operators that are multiplets under the  $(S_3)^3$  symmetry, the coefficient of the largest component is given.

ficient of order  $h_b h_s$ . The two other operators that have the same coefficient,  $(Q_3 Q_3)(Q_i L_i)$  and  $(Q_i Q_i)(Q_3 L_i)$ , each involve third generation fields, and therefore contribute to proton decay at a rate that is further suppressed by small CKM angles. The coefficients for all four components of the leading operator are given by

$$\begin{aligned}
O = \frac{c}{2} \frac{h_b}{M_*} [ & h_s (Q_2 Q_2)(Q_1 L_1) - h_s \lambda (Q_1 Q_1)(Q_2 L_1) \\
& - \mathcal{O}(h_d)(Q_2 Q_2)(Q_1 L_2) + h_d (Q_1 Q_1)(Q_2 L_2) ].
\end{aligned} \tag{31}$$

The overall coefficient  $c$  is a number of  $\mathcal{O}(1)$ , and the factor of  $1/2$  has been included to compensate for the symmetry of each term under the interchange of either the two  $Q_1$ 's or two  $Q_2$ 's. The striking feature of this multiplet is that the operators involving first generation lepton fields  $L_1$  have larger coefficients than those involving second generation fields  $L_2$ .

The only operator involving right-handed fields that appears potentially dangerous is  $h_s(uc)(se - \lambda s\mu)$ . However, the contribution of this operator to proton decay is in fact negligible. Since all fields are right-handed, their flavor cannot change via  $W$ -exchange. A  $c$  squark can change to a  $u$  squark

$Z_2^{U,D}$	$Q_1, E_1$	$Q_2, E_2$	$Q_3, E_3$	$U_1$	$U_2$	$U_3$	$D_1, L_1$	$D_2, L_2$	$D_3, L_3$	$H_U$	$H_D$
	+	+	+	+	-	-	+	-	-	-	-

Table 4: Charge assignments of MSSM fields under the  $Z_2^{U,D}$  symmetry.

via a flavor off-diagonal element of the RR squark mass matrix, and the resulting operator can be dressed by a gluino. Since the off-diagonal entry is  $(m_{RR}^2)_{12}/m^2 \simeq h_c^2 \lambda$ , the effective coefficient is smaller than  $h_s h_c$ . Thus, this operator is negligibly small compared to the leading left-handed operator discussed above. The same can be shown for all the remaining operators in Table 3.

What we have concluded based on the leading operator is that our model favors proton decay to  $\nu_e$  and  $e$  over decay to  $\nu_\mu$  and  $\mu$ . This result is in striking contrast to the situation in grand unified theories, and provides a counterexample to a common theoretical prejudice. Usually it is assumed that the operator involving  $L_1$  should be most strongly suppressed because it only violates the electron's U(1) chiral symmetry through the tiny electron Yukawa coupling. Nevertheless, the operator involving the electron in our model is larger than the operator involving the muon.

This unusual feature can easily be understood by considering the residual  $Z_2^{U,D}$  symmetry that is present when the Yukawa couplings of first generation fields are set to zero. Suppose that the all the breaking parameters except  $h_u$  and  $h_d$  are present. This leaves a  $Z_2^{U,D}$  symmetry, where the fields have the charge assignments given in the Table. 4.  $Z_2^{U,D}$  is a diagonal subgroup of  $Z_2^U$  and  $Z_2^D$ . What we learn from this table is that the first generation  $L$  field is even under this  $Z_2$ , while the second and third generation  $L$  fields are odd. Thus, in the symmetry limit,  $Z_2^{U,D}$  forbids the dimension-five operators containing  $L_2$  but allows those involving  $L_1$ . The same argument forbids operators involving  $L_3$  in the  $Z_2^{U,D}$  symmetry limit.

The predicted nucleon decay modes are obtained from Eq. (31) by ‘dressing’ the two-scalar-two-fermion operators with wino exchange.<sup>¶</sup> Below, we

---

<sup>¶</sup>In principle, one can dress the operator  $uds\nu$  by gluino exchange. Note that this operator is an SU(3) flavor singlet because of the total anti-symmetry in the color indices. The gluino dressing can possibly lead to the four-fermion operator  $(ud)(s\nu)$ ,  $(ds)(u\nu)$  or  $(su)(d\nu)$ . However, all these operators have mixed symmetry under flavor SU(3). Because of the high degeneracy among the squarks in our model, flavor SU(3) is a very good sym-

follow the notation of Hisano, Murayama and Yanagida [22]. The first operator  $ch_b h_s (Q_2 Q_2)(Q_1 L_1)/M_*$  gives us the following four-fermion operators<sup>||</sup>

$$\mathcal{L} = \frac{\alpha_W}{2\pi} \frac{ch_b h_s}{M_*} [-\lambda(du)(s\nu_e) - \lambda(su)(d\nu_e)] (f(c, e) + f(c, d)). \quad (32)$$

where  $f$  is the “triangle diagram factor” [23], a function of the wino and scalar masses:

$$f(i, j) = \frac{M_2}{m_i^2 - m_j^2} \left( \frac{m_i^2}{m_i^2 - M_2^2} \ln \frac{m_i^2}{M_2^2} - \frac{m_j^2}{m_j^2 - M_2^2} \ln \frac{m_j^2}{M_2^2} \right). \quad (33)$$

and where parentheses now indicate the contraction of spinor indices. In the limit where  $M_2 \ll m$ , this function is well approximated by  $f \simeq M_2/m^2$ . The second term in (31),  $-ch_b h_s \lambda(Q_1 Q_1)(Q_2 L_1)/M_*$ , gives us the four-fermion operators

$$\mathcal{L} = \frac{\alpha_W}{2\pi} \frac{ch_b h_s}{M_*} [\lambda(du)(s\nu_e) + \lambda(su)(ue)] (f(c, e) + f(u, d)). \quad (34)$$

All the fields in (32) and (34) are in the mass eigenstate basis, and terms of higher orders in  $\lambda$  have been neglected. The sum of these results is given by

$$\mathcal{L} = \frac{\alpha_W}{2\pi} \frac{ch_b h_s}{M_*} [-\lambda(su)(d\nu_e) + \lambda(su)(ue)] (f(c, e) + f(c, d)). \quad (35)$$

Here we have used the fact  $f(c, d) = f(u, d)$  to good accuracy. Note that there is a precise cancellation between  $(du)(s\nu_e)$  operators in (32) and (34). This has an important effect on the relative branching ratio between the charged lepton and neutrino mode, as we will discuss below.

The ratios of decay widths can be estimated using the chiral Lagrangian technique [24, 25, 22]. We find

$$\Gamma(p \rightarrow K^+ \bar{\nu}_e) : \Gamma(p \rightarrow K^0 e^+) : \Gamma(n \rightarrow K^0 \bar{\nu}_e)$$

metry even with the squarks. Therefore, the four-fermion operators are suppressed by the small non-degeneracy among the squarks, and hence are negligible. There is a possibility that gluino dressing of operators involving third-generation fields may be important. But their coefficients are already as small as the one we discuss here, and they need to pick up smaller mixing angles and hence are negligible.

<sup>||</sup>In the following discussion, we assume that the Cabibbo mixing originates from the down sector, i.e.  $a = 1$ ,  $a' = 0$  in eq. (3) and (4). However we checked that all the results remain the same even when the Cabibbo mixing comes from both the down and the up sectors, or even solely from the up sector.



$$= \left| \frac{2m_p}{3m_B} D \right|^2 : \left| 1 - \frac{m_p}{m_B} (D - F) \right|^2 : \left| 1 - \frac{m_n}{3m_B} (D - 3F) \right|^2 = 0.4 : 1 : 2.7. \quad (36)$$

where we have taken  $m_B \equiv 1150 \text{ MeV} \simeq m_\Sigma \simeq m_\Lambda$ ,  $D = 0.81$  and  $F = 0.44$ . We have stressed earlier that the dominance of the electron over the muon mode is a remarkable feature of this model, which can never happen in SUSY-GUTs [22, 26, 27]. In addition, what is remarkable about the result in (36) is that the proton's charged lepton decay mode dominates over the neutrino mode. This is a consequence of the cancellation of the  $(du)(s\nu_e)$  operator in (35). The dominance of  $p \rightarrow K^0 e^+$  over  $p \rightarrow K^+ \bar{\nu}_e$  is rarely the case in SUSY-GUTs.

Finally, we come to the overall rate. We find

$$\begin{aligned} & \tau(n \rightarrow K^0 \bar{\nu}_e) \\ &= 4 \times 10^{31} \text{ years} \left| \frac{1}{c} \frac{0.003 \text{ GeV}^3}{\xi} \frac{0.81}{A_S} \frac{5}{(1 + \tan^2 \beta)} \frac{\text{TeV}^{-1}}{f(c, e) + f(c, d)} \right|^2 \end{aligned} \quad (37)$$

This result includes the effect of running the dimension-five operator between the Planck scale and  $m_Z$ , (a factor of  $A_S = 0.81$  in the amplitude if  $m_t = 175 \text{ GeV}$ ,  $\tan \beta = 2$ ,  $\alpha_s(m_Z) = 0.12$ ) and between  $m_Z$  and  $m_n$  (a factor of 0.22 in the amplitude). In the expression above,  $\xi$  is the hadronic matrix element of the four-fermion operator evaluated between nucleon and kaon states; its exact value is rather uncertain, but is estimated to be within the range  $\xi = 0.003\text{--}0.03 \text{ GeV}^3$ . If we take that  $M_2 \sim 100 \text{ GeV}$  and  $m_{\tilde{q}} \sim 700 \text{ GeV}$ , and  $m_{\tilde{l}} \sim 300 \text{ GeV}$ , then the triangle functions  $f(c, e) + f(c, d) \sim (1.8 \text{ TeV})^{-1}$ . Thus, if  $c = 1$  and  $\xi = 0.003 \text{ GeV}^3$ , we obtain a mean lifetime  $12.7 \times 10^{31}$  years, which can be compared to the experimental bound,  $\tau(n \rightarrow K^0 \bar{\nu}_e) > 8.6 \times 10^{31} \text{ years}$ . It is interesting to note that the coefficient  $4 \times 10^{31}$  in (37) would be the same in the minimal SU(5) SUSY-GUT with an extremely large color-triplet Higgs mass  $M_{H_C} = 10^{17} \text{ GeV}$ . Thus, the rate in our model is roughly comparable. Overall, the  $(S_3)^3$  symmetry gives us just enough suppression of dimension-five operators to evade the current bounds, so the model is phenomenologically viable. Since the SuperKamiokande experiment is expected to extend Kamiokande's current reach by another factor or 30, there is a very good chance that the  $n \rightarrow K^0 \bar{\nu}_e$  mode may be seen. It is an exciting prediction of this model that the  $p \rightarrow K^0 e^+$  and  $K^+ \bar{\nu}_e$  modes are

likely to be seen at the same time because their rates are close to each other, as we saw in eq. (36).

## 7 Conclusions

We have shown in this paper how to incorporate the leptons in the  $(S_3)^3$  model presented in Ref. [10]. By assuming that all possible nonrenormalizable operators are generated at the Planck scale, we showed that the transformation properties of both the leptons and the flavor symmetry breaking fields could be uniquely determined. We then demonstrated that the phenomenological constraints from lepton flavor violation, CP violation, and proton decay are indeed satisfied in our model. The most striking prediction that emerged from our analysis is the dominance of proton decay to final states involving first generation lepton fields, unlike the case in SUSY GUTs. We showed that the ratios of decay widths for the largest modes  $n \rightarrow K^0 \bar{\nu}_e$ ,  $p \rightarrow K^+ \bar{\nu}_e$ , and  $p \rightarrow K^0 e^+$  are approximately  $0.4 :: 1 :: 2.7$ . Given our estimate of the total rate, we pointed out that all three modes may be within the reach of the SuperKamiokande experiment and could well be discovered simultaneously.

## Acknowledgments

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-90-21139.

## References

- [1] S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193**, 150 (1981).
- [2] Y. Nir and N. Seiberg, *Phys. Lett.* **B309**, 337 (1993).
- [3] M. Leurer, Y. Nir and N. Seiberg, *Nucl. Phys.* **B420**, 468 (1994).
- [4] M. Dine, R. Leigh and A. Kagan, *Phys. Rev.* **D 48**, 4269 (1993).

- [5] L.E. Ibáñez and G.G. Ross, *Phys. Lett.* **B332**, 100 (1994);  
P. Binetruy and P. Ramond, *Phys. Lett.* **B350**, 49 (1995);  
V. Jain and R. Shrock, *Phys. Lett.* **B352**, 83 (1995); ITP-SB-95-22,  
hep-ph/9507238;  
Y. Nir, *Phys. Lett.* **B354**, 107 (1995);  
E. Dudas, S. Pokorski and C.A. Savoy, *Phys. Lett.* **B356**, 45 (1995);  
M.M. Robinson and J. Ziabicki, EFI-95-50, hep-ph/9508322;  
T. Gherghetta, G. Jungman and E. Poppitz, UM-TH-95-27, hep-  
ph/9511317.
- [6] E. Dudas, S. Pokorski and C.A. Savoy, SACLAY-DPHT-95-094, hep-  
ph/9509410.
- [7] P. Pouliot and N. Seiberg, *Phys. Lett.* **B 318**, 169 (1993).
- [8] A. Pomarol and D. Tommasini, CERN-TH/95-207; hep-ph/9507462.
- [9] D.B. Kaplan and M. Schmaltz, *Phys. Rev.* **D49**, 3741 (1994).
- [10] L.J. Hall and H. Murayama, *Phys. Rev. Lett.* **75** 3985 (1995).
- [11] S. Coleman, *Nucl. Phys.* **B310**, 643 (1988);  
S. Giddings and A. Strominger, *Nucl. Phys.* **B307**, 854 (1988);  
G. Gilbert, *Nucl. Phys.* **B328**, 159 (1989).
- [12] Y. Kawamura, H. Murayama, and M. Yamaguchi, *Phys. Rev.* **D51**  
(1995) 1337;  
H. Murayama, invited plenary talk given at 4th International Confer-  
ence on “Physics Beyond the Standard Model,” Lake Tahoe, CA, 13-18  
Dec 1994, Proceedings edited by J.F. Gunion, T. Han, and J. Ohnemus,  
World Scientific, 1995. hep-ph/9503392.
- [13] H. Murayama and D.B. Kaplan, *Phys. Lett.* **B336**, 221 (1994).
- [14] N. Sakai and T. Yanagida, *Nucl. Phys.* **B197**, 533 (1982).
- [15] L.E. Ibáñez and G.G. Ross, *Nucl. Phys.* **B368**, 3 (1992).
- [16] C.D. Carone and H. Murayama, *Phys. Rev. Lett.* **74**, 3122 (1995);  
*ibid.*, *Phys. Rev.* **D52**, 484 (1995).

- [17] C.D. Carone and H. Murayama, *Phys. Rev.* **D52**, 4159 (1995).
- [18] H. Georgi and C. Jarlskog, *Phys. Lett.* **86B**, 297 (1979).
- [19] R. Barbieri, L. Hall, and A. Strumia, *Nucl. Phys.* **B445** 219 (1995).
- [20] S. Dimopoulos and L.J. Hall, *Phys. Lett.* **B344**, 185 (1995).
- [21] E. Gabrielli, A. Masiero, L. Silvestrini, ROME1-1109-95, Sep 1995, hep-ph/9509379.
- [22] J. Hisano, H. Murayama, T. Yanagida, *Nucl. Phys.* **B402**, 46 (1993).
- [23] P. Nath and R. Arnowitt, *Phys. Rev.* **D38**, 1479 (1988).
- [24] M. Claudson, M.B. Wise and L.J. Hall, *Nucl. Phys.* **B195**, 297 (1982).
- [25] S. Chadha and M. Daniel, *Nucl. Phys.* **B229**, 105 (1983).
- [26] A. Antaramian, LBL-36819, Feb 1995.
- [27] K.S. Babu, S.M. Barr, BA-95-21, Jun 1995, hep-ph/9506261.